The GBT Sensitivity Calculator User's Guide

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1. Introduction to the GBT Sensitivity Calculator

The new GBT Sensitivity Calculator has been designed to provide observers an easy way to determine the time needed to complete a proposed project or the expected sensitivity achieved by a project of a given length. In comparison with our previous calculator, the replacement is significantly more sophisticated and leads users through a complex web of decisions and choices astronomers make as they think out their sensitivity estimates. The new calculator should simplify the writing of a proposals technical justification since a user can save the input parameters and results to a file which can then be attached to a proposal. Since an attachment will contain very complete details, it will also reduce the chance that a reviewer will misinterpret the technical justification.

The calculator has been designed to satisfy 95% or so of 'traditional' spectral line, pulsar and continuum observations and is not guaranteed to produce results better than 10% in sensitivity or 20% in time. It is up to the user on how best to derive from the calculator's output the total time for their sessions and projects. Thus, we expect those writing proposals to continue to use the technical justification section of their proposals to describe how the results of the calculator were used to derive the time estimates for their projects. Users can also include in their technical justifications as many output logs from the calculator as they feel are needed.

This document has been compiled to guide users through the flow of questions asked by the calculator in order to produce a final sensitivity or time estimate which can then be included in the technical justification of the proposal. The structure and common features of the sensitivity calculator can be found in Chapter 2. The following chapters are devoted to separate frames within the sensitivity calculator that require user input and should each be completed sequentially.

3. General Information
4. Hardware Information
5. Source Information
6. Data Reduction
7. Results

The Appendix provides a detailed discussion of the parameters and equations used by the calculator.

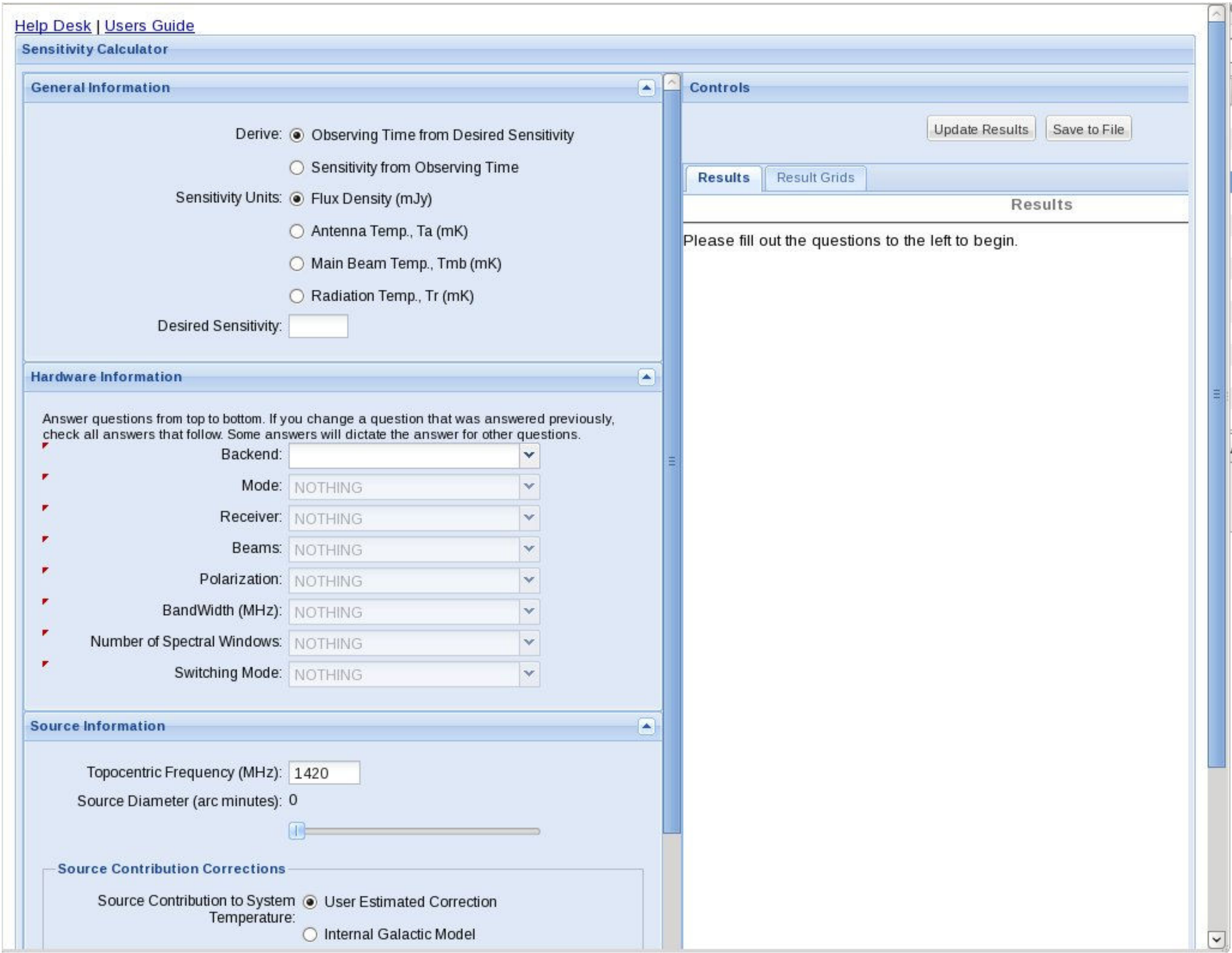
1.1 How Start the GBT Sensitivity Calculator

The Sensitivity Calculator is located at https://dss.gb.nrao.edu/calculator-ui/war/Calculator_ui.html. You will need to supply your 'My NRAO' username and password.

2. Structure and Common Features of the Sensitivity Calculator

When you first start the GBT Sensitivity Calculator you will be presented with the screen shown in Figure 2.1.

Figure 2.1: The GBT Sensitivity Calculator



2.1 Structure

The GBT Sensitivity calculator has been designed in a way that guides the user through various sequential steps. In general you should start by completing boxes from top to bottom, as the questions asked of you will depend on your answers to previous questions. For example, you will not be asked how many beams you wish to use if you have selected a single beam receiver. Any questions that are not applicable to your proposed observations or that cannot be asked at an early stage will be grayed out. As you select your answer to a question, the calculator will automatically update and if necessary, further questions may be asked of you.

Questions asked of the user are organized by topics given in the frames on the left hand side of the screen (General Information, Hardware Information, Source Information and Data Reduction). At any time you may press the 'Update Results' button to view any parameters that the calculator can return to you based on the answers you have given. If the calculator is not able to return a derived sensitivity or total observing time then you still have some questions to answer.

2.2 Features

- Help Desk – You may select this link to send an email to the Dynamic Scheduling System (DSS) helpdesk using your default email application.
- – Press the up arrow button to minimize its associated frame.
- – Press the down arrow button to expand its associated frame.
- – Red triangles denote questions that must be answered in order for the calculator to proceed or denote fields that have been altered, requiring you to press the 'Update Results' button on the right side of the calculator.

3. General Information Frame

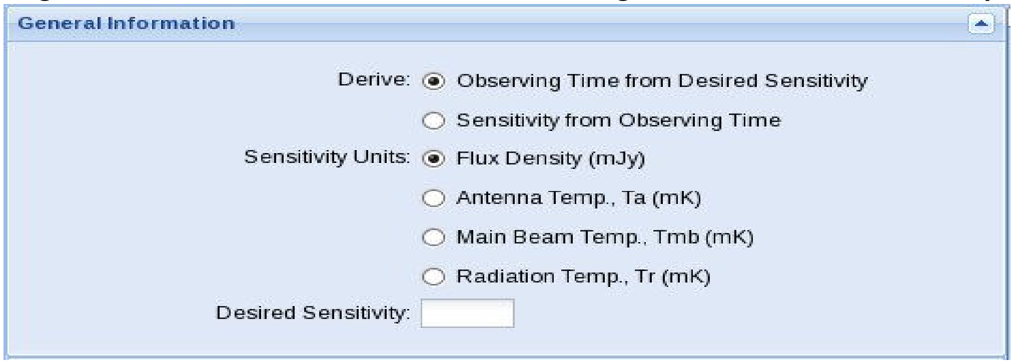
You must select whether you wish to derive the total observing time from a given sensitivity or vice versa. In either case, you must first need to choose your units for sensitivity. The allowed units are:

- **Flux Density (mJy) -** $(10^{-29}$ Watts $\mathrm{m}^2\mathrm{Hz}^{-1})$, and as if measured from above the Earth's atmosphere (Default).
- **Antenna Temp., Ta (mK) -** as measured below the Earth's atmosphere.
- **Radiation Temp., Tr (mK)** - as if measured from above the Earth's atmosphere and defined for sources of any size.
- **Main Beam Temp., Tmb (mK)** – similar to Tr, but defined for sources whose diameter extends to the first nulls in the telescope's beam.

3.1 Observing Time from Desired Sensitivity

Enter the sensitivity that you wish to achieve in the Desired Sensitivity box in your chosen units.

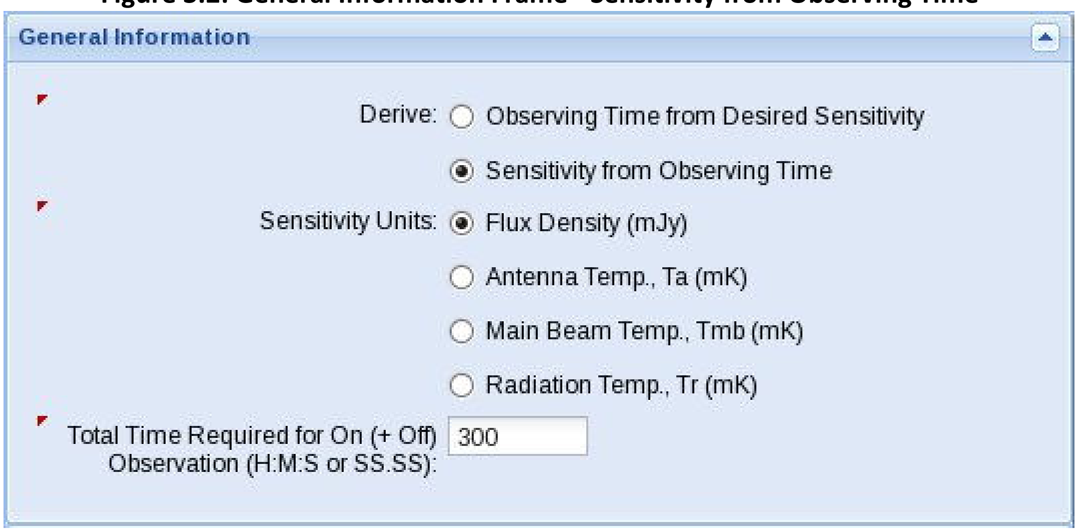
Figure 3.1: General Information Frame - Observing Time from Desired Sensitivity



3.2 Sensitivity from Observing Time

Enter the total time required for your observations. Units of time can be entered in seconds or sexegesimal format.

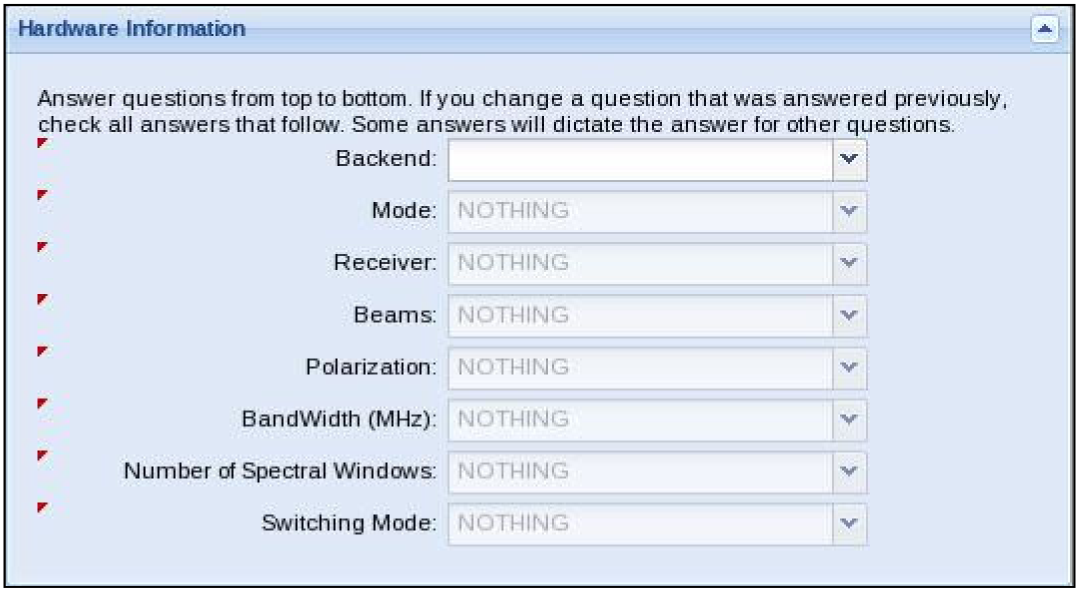
Figure 3.2: General Information Frame - Sensitivity from Observing Time



4. Hardware Information Frame

You will be asked a series of questions concerning your choice of hardware and the calculator will check the answers to make sure that they can be accommodated by the hardware. Only those questions needed for the sensitivity calculator will be asked. For example, if you select MUSTANG as your backend, then all subsequent choices will be filled out for you with the MUSTANG default values, and you will not be required (or able) to answer any further questions within this frame.

Figure 4.1: Hardware Information Frame



The available fields within the Hardware Information Frame are:

- **Backend** – You may currently choose from the following backends:
  - Caltech Continuum Backend (CCB)
  - GBT Digital Continuum Receiver (DCR)
  - GBT Spectrometer
  - Green Bank Ultimate Pulsar Processor (GUPPI)
  - Mustang
  - Spectral Processor
  - VErsitile GB Astronomical Spectrometer (VEGAS)
  - Zpectrometer

- **Mode –** This will be selected automatically depending on your choice of backend. The available modes currently available for each backend are:
  - Continuum – CCB, DCR, VEGAS and Mustang
  - Spectral Line – VEGAS, GBT Spectrometer, Spectral Processor and Zpectrometer
  - Pulsar – VEGAS, GUPPI

- **Receiver** – If you have selected VEGAS, DCR, GBT Spectrometer or Spectral Processor as your backend then you will need to select which GBT receiver your wish to use.

- **Beams** – If the receiver you have selected is capable of using more than one beam then you will be prompted to enter how many beams you wish to use here.

- **Polarization** - You will be given a choice between Dual and Full for the GBT Spectrometer and FPGA Spectrometer, and between Cross_QU, Dual and Full with the Spectral Processor.

- **Bandwidth** – You may be given a choice of available bandwidths depending on which backend you have selected.

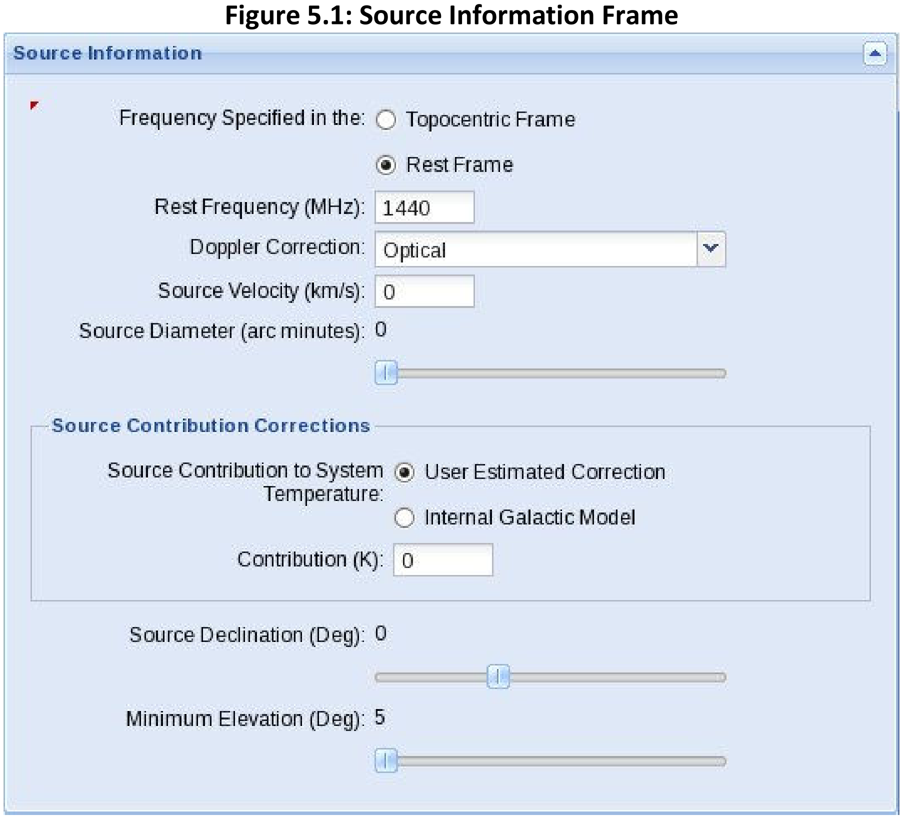
- **Spectral Windows** – You may be given a choice of how many spectral windows you wish to use for your observations.

- **Switching Mode** – Currently you may choose between 'In-Band Frequency Switching', 'Out-of-Band' Frequency Switching' and 'Position Switching'.

5. Source Information Frame

Since projects usually observe multiple sources, you may want to run the sensitivity calculator for each, but it is also acceptable to run the calculator for a representative source, or a representative set of source parameters. If the latter approach is taken then you will need to infer how that calculation for a single or representative source translates into the time or sensitivity requirements for the body of your sources.

Figure 5.1: Source Information Frame



User input is required for the following fields:

- **Topocentric Frame / Rest Frame** – You will be asked if the observing frequencies will be given in the line's reset frame or in the topocentric frame. The default is 'Rest Frame'

- **Topocentric / Rest Frequency (MHz)** – The default will be the middle of the selected receivers band.

- **Doppler Correction** – If you had previously selected 'Rest Frame' then you will be asked to select between an optical, radio and redshift Doppler correction. The default is 'Optical'.

- **Source Velocity (km/s)** / **Redshift** - Depending on the selected Doppler correction you will be asked to supply a representative source velocity or redshift. The default in either case is zero.

- **Source Diameter (arc minutes)** – Use the slider to set a representative size for your source. Available values range from 0 (point source) to the FWHM of the GBT beam. The calculator assumes the source brightness distribution is that of a uniform disk. The slider is not available is you have selected units of Ta or Tmb as the definitions of these conventions include a predefined source size.

- **Source Contribution Corrections** – You will be asked whether to apply a correction for the representative source's continuum level to the calculator's system temperature calculations. The options are:

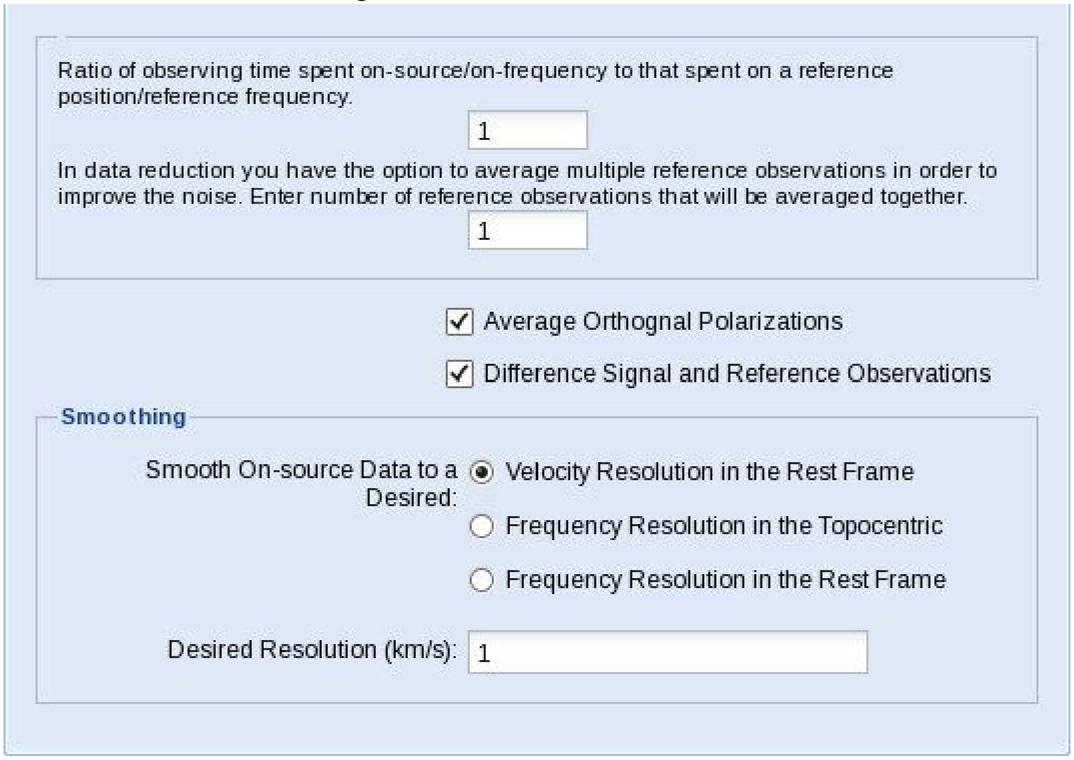
  - **User Estimated Correction** – If you select this options then you will be asked to provide the background level in the units that you have selected in the 'General Information' frame.
  - **Internal Galactic Model** – If selected then the calculator will ask for a representative J2000 Right Ascension of the source. The calculator will then augment the system temperature with the approximate continuum background from the Milky Way for the specified position using the 408 MHz survey of Haslam et al (1981, A&A, 100, 209)

- **Source Declination (Deg)** – Use the slider to set a representative source Declination in degrees (default = 0°).
- **Minimum Elevation (Deg)** – Use the slider to set the minimum allowable elevation for observations of the source (default = 5°). The calculator will provide a suggested minimum elevation, dependent on your source declination and observing frequency, if the value you entered is less than the recommended value.

6. Data Reduction Frame

Observers have a number of choices in how they collect and reduce their data that significantly affect the time they will need for an experiment and the corresponding sensitivity they will achieve. Only those that are most common have been included in the calculator.

Figure 6.1: Data Reduction Frame



The available fields available for data reduction depend upon your answers in the hardware frame. Possible questions include:

- **Ratio of observing time spent on-source/on-frequency to that spent on a reference position/reference frequency** – If you have selected any switching mode other than total power, then you will be asked:
  - The ratio of time spent on your signal (on-source or on-frequency) observation to their reference observation. The default is set to 1.
  - The number of reference observations that will be averaged together. The default is set to 1.

- **Average Orthogonal Polarizations** – You will be offered this choice based on your hardware configuration. If applicable then this box will be checked by default.

- **Difference Signal and Reference Observations** – You will be asked whether you plan on differencing signal and reference observations. By default this is set to on.

- **Smoothing** – For spectral line observations you will be required to provide how you will smooth both the on-source and off-source data and whether you will be smoothing to a specified:
  - *Velocity Resolution* (km/s) in the source's rest frame (default)
  - *Frequency Resolution in the Topocentric* Frame (MHz)

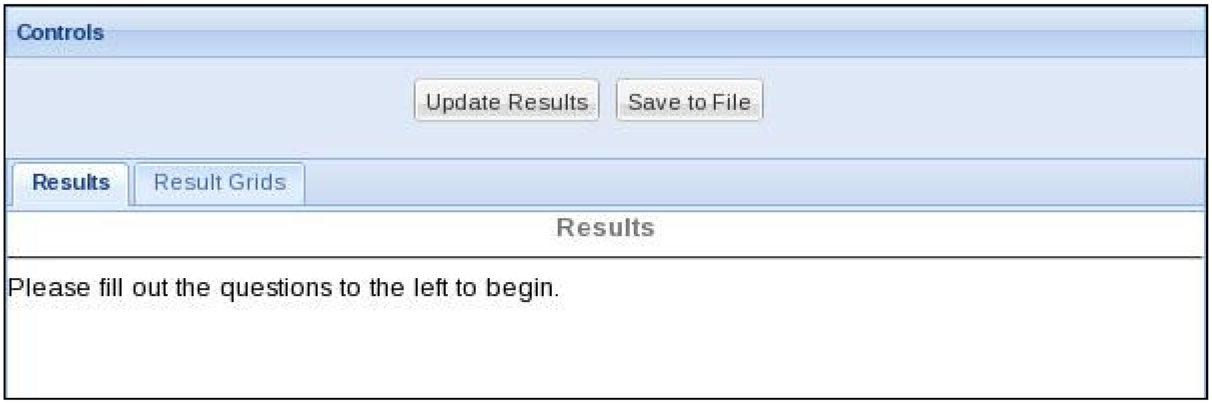
- *Frequency Resolution in the Rest Frame* (MHz)

- **Resolution** – Enter the desired resolution in the units selected above (Default = 1).

7. Results Frame

The results frame on the calculator's right side is where all the output from the sensitivity calculator will be displayed. Figure 7.1 shows the results frame prior to any entries by the user.

Figure 7.1: A Blank Results Frame



7.1 Controls

The only interface with the results frame is through the two buttons under 'Control'.

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 - Press the 'Update Results' button to display all results that the sensitivity calculator can currently return based on the information you have given. The calculator will also prompt you to press this button if you have changed any of the entries in the user input frames on the left of the screen by displaying a red arrow next to any field that has been altered.

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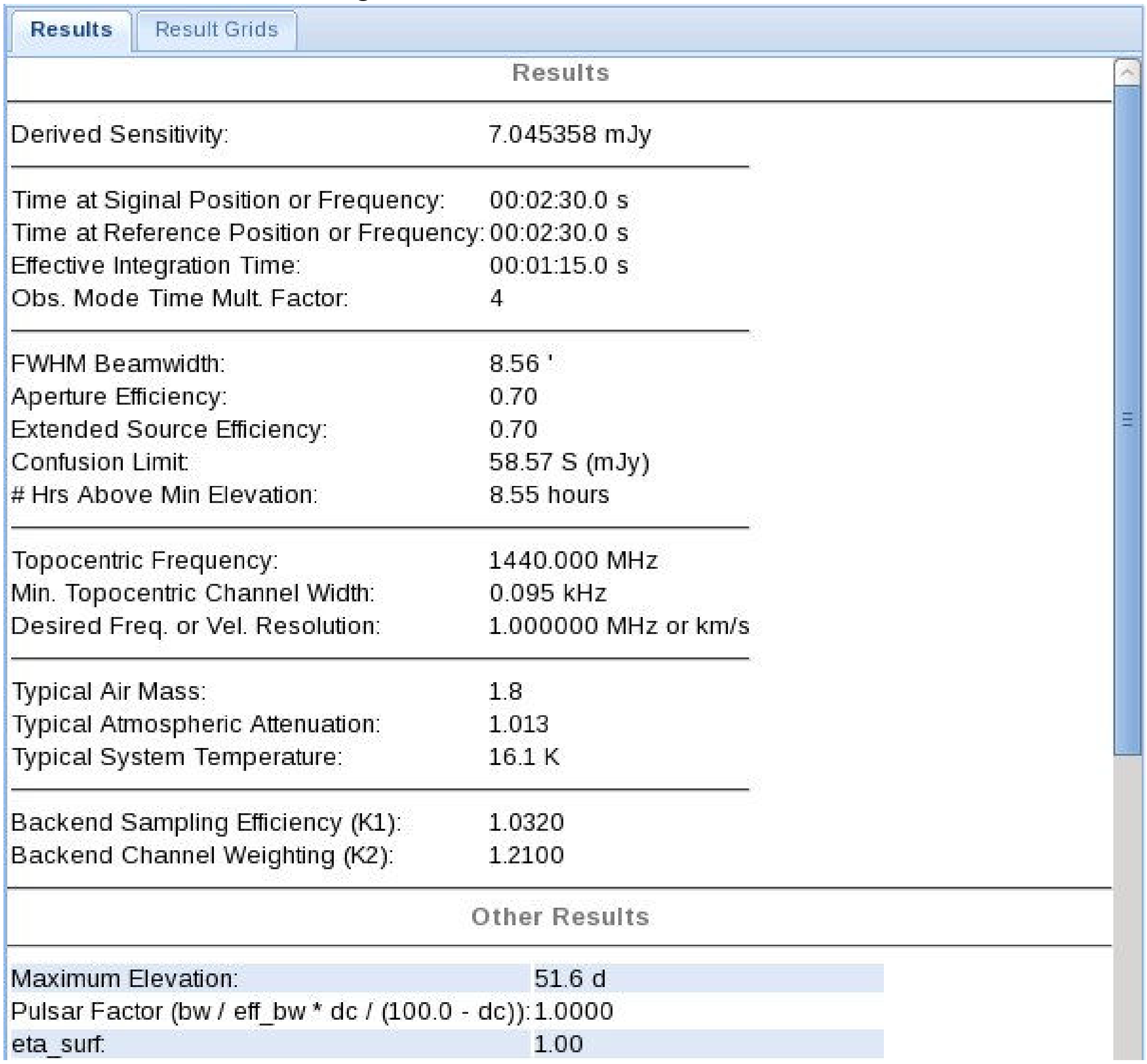
 - If you press the 'Save to File' button then you will be given the option of saving the information displayed in the results frame as a text file. The file can then be attached to your proposal.

7.1 The Results Window

The results grid is for debugging as it presents all of the internal variables and values used by the calculator's underlying code. The results tab provides the most user friendly way to look at your results. Figure 7.2 gives an example of the 'Results' tab layout after all information has been provided by the user.

Figure 7.2: Results Frame – 'Results' Tab



Outputs from the sensitivity calculator in this mode are given under the following headings:

- **Results** – This section contains information that will be of use to you when writing your technical justification. If you are unable to derive a final sensitivity or total observing time, then you will need to include additional information in the user input frames to the left of the screen (and also remember to press the 'Update Results' button). You may also find it helpful to look under the 'User Input' heading (Also in the results frame) and scan it for any fields which appear to be blank, in which case you can enter that missing information in the relevant frame.

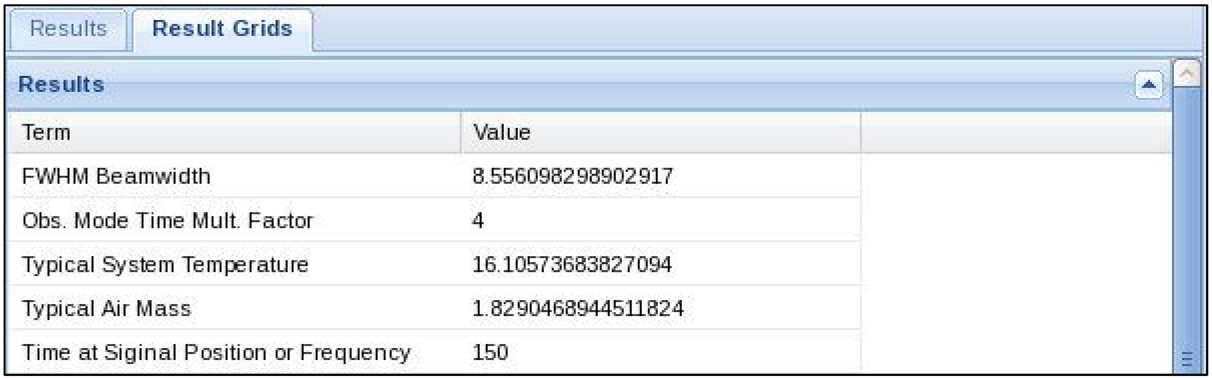
- **Messages** – Miscellaneous information that the calculator may deem important will be displayed under this section.

- **Other Results** – The values of constants used for the sensitivity calculators calculations. Details of these constants and algorithms can be found in the Appendix.

- **User Input** – All of the information entered into the General Information, Hardware Information, Source Information and Data Reduction frames will be displayed under this heading.

7.1.1 The Grid Results Tab

Figure 7.3: Results Frame – 'Result Grids' Tab



Term	Value
FWHM Beamwidth	8.556098298902917
Obs. Mode Time Mult. Factor	4
Typical System Temperature	16.10573683827094
Typical Air Mass	1.8290468944511824
Time at Siginal Position or Frequency	150

If you select the 'Result Grids' tab, then the output will be displayed in table format (Figure 7.3). Unlike the 'Results' tab, there are only two headings:

- **Results** – Contains all of the parameters that are given under the headings 'Results' and 'Other Results' in the Result Tab.

- **User Input** – All of the information entered into the General Information, Hardware Information, Source Information and Data Reduction frames will be displayed under this heading.

Appendix

A. Definition and Terms

Term	Definition
A	Representative atmospheric attenuation, $e^{\tau \cdot AirMass}$, for the typical weather condition and elevation of an observation at the user's observing frequency
AirMass	Representative air mass through which the observations will be made
BW	Bandwidth in MHz, either native to the backend or the bandwidth to which the user smoothes
BW_{Ref}	Bandwidth in MHz that the reference (off) observation will be smoothed to
c	Speed of light in m s^{-1}
DishRadius	Illumination radius for the selected receiver in m
Δf_{REST}	Frequency resolution in the rest frame in MHz
ΔV_{REST}	Velocity resolution in the rest frame in m s^{-1}
El_{Min}, El_{Max}	Range in observing elevation in °
EST	Effective system temperature in K ($T_{sys} \cdot e^{\tau \cdot AirMass}$)
EST_0	Effective system temperature under the best possible weather conditions

EST_{TS}	Effective system temperature but augmented by the expect loss in efficiency due to tracking and surface errors
FeedTaper	Feed taper illumination of the reflector in dB.
Frequency	Topocentric frequency in MHz
FWHM	Full-width, half-maximum beam width in ‘
η_{MB}, η_{l} η_{fss}, and η_{S}	Efficiency which takes into consideration surface errors as a function of frequency and source size for various intensity conventions.
η_0	Aperture efficiency at long wavelengths and receiver and frequency dependent
η_A	Aperture efficiency which takes into consideration surface errors as a function of frequency and elevation
η_{Dss}	Normalized observing efficiency, in units of time, as suggested by DSS simulations and the product η_{Track} η_{Surf} η_{Atm}
η_{Track}, η_{Surf}, η_{Atm}	The normalized observing efficiency, in units of time, as suggested by DSS simulations due to tracking errors (e.g., wind induced), thermal-induced surface errors, and atmospheric conditions.
θ_{Source}	Source size in ’
k	Boltzmann’s constant
K_1	Sampling sensitivity of a backend and, thus, is hardware dependent
K_2	Autocorrelation channel weighting factor for spectral line backends or a measure of the independence of samples for continuum observing; hence hardware dependent
$N_{RefAvrg}$	Number of references observations that will be averaged together in the data reduction and used for every signal (on) observation
$N_{RefSmthAvrg}$	Amount by which a reference observation is smoothed or the number of reference observations that are averaged together
$N_{Samp}^{\mathrm{Uncorr}}$	Degree to which backend inputs measure uncorrelated signals. (For example, = 2 when averaging orthogonal polarizations or when using nodding or in-band frequency switching)
R_{SigRef}	Ratio of time spent on the signal observation to the time on the reference observatio
σ_S	Sensitivity in units of Jy (above atmosphere)
$\sigma_{T_R^*}$	Sensitivity in K in the T_R^* (above atmosphere) temperature scale
σ_{T_A}	Sensitivity in K in the T_A (below atmosphere) temperature scale
τ	Representative zenith atmospheric opacity in units of Nepers
τ_0	Best possible zenith atmospheric opacity in units of Nepers
t_{eff}	Effective integration time in s - essentially the time that satisfies the radiometer equation and is related to the actual observing time in ways that depend upon the observing tactics
t_{Sig}	Time spent in s on a source or signal position or frequency
t_{Ref}	Time spent in s on a reference position or frequency
t_{Total}	Total time in s needed to complete an observation
T_{Atm}	Temperature of the atmosphere in K to use in an estimate of T_{Sys} -- approximate temperature of the atmospheric layer that is contributing most to the opacity
T_{CMB}	The cosmic microwave background = 2.7 K
T_{BG}	Either the continuum temperature in K of the user’s source or the temperature in K of the galactic background in the direction of the observation
T_{Rcvrl}	Contribution to T_{Sys} in K from the receiver
T_{Sys}	Expected approximate system temperature in K at a representative elevation
V	Source velocity in m/s
z	Redshift (V/c)

B. Basic Equations

The algorithms used by the sensitivity calculator are a modified version of the classic radiometer equation. The modified equation describes the theoretical noise one would obtain if observing above the atmosphere with a given effective system temperature (EST_{TS} ; see §E.1 for the relationship between EST_{TS} and T_{SYS}) and bandwidth (BW in MHz) for a given effective duration of an observation (t_{eff} in sec). If we ignore the contribution from 1/F noise:

$$\sigma_{T_R^*} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss}}\right) \frac{1}{\sqrt{N_{Samp}^{\text{Uncorr}} \cdot 10^6 \cdot BW \cdot t_{\text{eff}}}}$$

N_{Samp}^{Uncorr} depends upon the details of the observing tactics (§E.5). K_1 = the sampling sensitivity of a backend (§E.3).

The calculator uses the definition of the T_R^* temperature scale of Kutner and Ulich (1981). Accordingly, T_R^* is the brightness temperature of an equivalent black-body source with the assumption that the source is a circular disc of uniform brightness. $\eta_l\ \eta_{fss}$ corrects for both the efficiency of the telescope's optics as well as the convolution of the source brightness distribution with the telescope's beam (§E.4).

Due to the rough accuracy needed, the calculator makes some simplifying assumptions:

- The Rayleigh-Jeans approximation holds at all frequencies,
- The ohmic-loss efficiency factors, η_r in Kutner and Ulich is taken to be equal to 1.

One should not use the results of the calculator for the accurate calibration of astronomical measurements.

Observers will be specifying (or requesting) sensitivities in the following units:

- Flux density (S in Jy = 10^{-26} Watts m^{-2} s^{-1}), as if observed from above the Earth's atmosphere
- Temperature as defined by the T_A temperature scale, as observed from the surface of the Earth
- Temperature as defined by the T_R^* temperature scale, as if observed from above the Earth's atmosphere
- Temperature as defined by the T_{MB} temperature scale, which is identical to the definition of T_R^* but with the assumption that the source size is the same as the telescope's beam to its first null. Here, $\eta_l\ \eta_{fss}$ is equal to the standard definition of beam efficiency, η_{MB}.

With the above simplifying assumptions, and the standard conversions between these units of intensity, the relationships of the theoretical noise for the various intensity scales are:

$$\sigma_{T_A} = \frac{\eta_S \cdot \pi \cdot DishRadius^2}{2k \cdot e^{\tau \cdot AirMass}} \cdot \sigma_S$$

$$\sigma_{T_A} = \frac{\eta_l \eta_{fss}}{e^{\tau \cdot AirMass}} \cdot \sigma_{T_R^*}$$

$$\sigma_{T_A} = \frac{\eta_{MB}}{e^{\tau \cdot AirMass}} \cdot \sigma_{T_{MB}}$$

$e^{\tau \cdot AirMass}$, hereafter called AtmosAtten, is the representative atmospheric attenuation for the elevation range of the observation; k = Boltzman's constant. Section E.4 describes how η_S, η_{MB} and $\eta_l\ \eta_{fss}$ depend upon the efficiency of the telescope's optics and source size.

Most observing tactics require differencing observations of a signal/source position (or signal frequency) with a reference position (or reference frequency). Since both the signal and reference observations have noise, the resulting difference will be noisier than either the signal or reference observation. Since t_{eff} is related to the noise in the resulting difference, t_{eff} is always less than the total duration of the signal plus reference observation.

Often one may spend different amount of time on each phase of an observations. For example, to save overhead it's common practice to use one reference observation for multiple signal observations. The calculator defines R_{SigRef} , which has a default value of 1, as the ratio of the time spent on each phase of an observation.

$$R_{\mathrm{SigRef}} = t_{\mathrm{Sig}}/t_{\mathrm{Ref}}.$$

To help reduce the extra noise from differencing observations, it's a common practice during data analysis to smooth the reference observation or to average multiple reference observations. The calculator defines $R_{RefSmthAvrg}$, which has the default value of 1, as the amount by which a reference observation is smoothed or averaged beyond any smoothing done to the signal observation. From the theory of the propagation of errors, the noise in the difference is:

$$\sigma^2 = \sigma_{Sig}^2 + \sigma_{Ref}^2$$

Assuming EST_{TS} is the same for the signal and reference observations, the above equation implies that:

$$\frac{1}{t_{\mathrm{eff}}} = \frac{1}{t_{\mathrm{Sig}}} + \frac{1}{R_{\mathrm{RefSmthAvrg}} \cdot t_{\mathrm{Ref}}}$$

And, therefore,

$$t_{\mathrm{eff}} = \frac{t_{\mathrm{Sig}} \cdot (R_{\mathrm{RefSmthAvrg}} \cdot t_{\mathrm{Ref}})}{t_{\mathrm{Sig}} + (R_{\mathrm{RefSmthAvrg}} \cdot t_{\mathrm{Ref}})}$$

The typical user isn't interested in the value of t_{eff}, t_{Sig} or t_{Ref}. Rather, users are mostly interested in the total duration needed for an observation (t_{Total}). If there is no overhead involved in the observing, then $t_{Total} = t_{Sig}+t_{Ref}$, and after a bit of algebra, one derives the following relationships between t_{eff} , t_{Total}, t_{Sig} and t_{Ref}:

$$t_{\mathrm{Total}} = t_{\mathrm{Sig}} + t_{\mathrm{Ref}} = t_{\mathrm{eff}} \cdot \frac{(R_{\mathrm{SigRef}} + R_{\mathrm{RefSmthAvrg}})(R_{\mathrm{SigRef}} + 1)}{R_{\mathrm{SigRef}} \cdot R_{\mathrm{RefSmthAvrg}}}$$

$$t_{\mathrm{eff}} = t_{\mathrm{Total}} \cdot \frac{R_{\mathrm{SigRef}} \cdot R_{\mathrm{RefSmthAvrg}}}{(R_{\mathrm{SigRef}} + R_{\mathrm{RefSmthAvrg}})(R_{\mathrm{SigRef}} + 1)}$$

$$t_{Sig} = t_{Total} \cdot R_{SigRef}/(R_{SigRef} + 1)$$

$$t_{Ref} = t_{Total}/(R_{SigRef} + 1).$$

The above radiometer equations for the various intensity scales, in terms of t_{Total}, are then:

$$\sigma_S = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_s \cdot \pi \cdot DishRadius^2}\right)\sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}}$$

$$\sigma_{T_R^*} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{ffs}}\right)\sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}}$$

$$\sigma_{T_{MB}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_{MB}}\right)\sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}}$$

$$\sigma_{T_A} = \left(\frac{K_1 \cdot EST_{TS}}{AtmosAtten}\right)\sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}}$$

Some observing tactics will not require taking a reference observation, which implies that $t_{\text{Total}}=t_{\text{eff}}$. In these cases:

$$\sigma_S = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_S \cdot \pi \cdot DishRadius^2}\right)\sqrt{\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}}$$

$$\sigma_{T_R^*} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss}}\right)\sqrt{\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}}$$

$$\sigma_{T_R^*} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_{MB}}\right)\sqrt{\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}}$$

$$\sigma_{T_A} = \left(\frac{K_1 \cdot EST_{TS}}{AtmosAtten}\right)\sqrt{\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}}$$

Simple inversions of these equations allow one to derive a total time from a user-supplied sensitivity:

$$t_{\text{Total}} = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_S \cdot \pi \cdot DishRadius^2 \cdot \ \sigma_S}\right)^2 \left(\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}}}\right)$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss} \cdot \sigma_{T_R^*}}\right)^2 \left(\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}}}\right)$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_{MB} \cdot \sigma_{T_{MB}}}\right)^2 \left(\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}}}\right)$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\sigma_{T_A} \cdot AtmosAtten}\right)^2 \left(\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvrg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvrg}}}\right)$$

Without reference observations:

$$t_{\text{Total}} = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_S \cdot \pi \cdot DishRadius^2 \cdot \sigma_S}\right)^2 \left(\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right)$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss} \cdot \sigma_{T_R^*}}\right)^2 \left(\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right)$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta_{MB} \cdot \sigma_{T_{MB}}}\right)^2 \left(\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right)$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\sigma_{T_A} \cdot AtmosAtten}\right)^2 \left(\frac{1}{10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right)$$

B. 1/F Gain Instabilities

All receivers have a sensitivity limit from 1/F gain instabilities. A few systems that were explicitly designed for continuum observations (e.g., Mustang and the Ka-Band receiver but only when used with the CCB backend) have much less of an issue with 1/F instabilities. These gain instabilities essentially place an upper limit on t_{Total}·BW – beyond a certain point, increasing the bandwidth or the amount of observing time does not improve one's sensitivity.

There are a number of tactics that lets one exceed the 1/F limit (e.g., averaging multiple short difference observations; making multiple maps with fast slewing). Since the calculator interface would need to be made much more complicated to capture all necessary details about the observing tactics, it cannot decide whether or not ones observations will actually exceed the limit. Therefore, to be on the safe side, the calculator warns a user whenever it is possible the observations will exceed the limit. If the user's planned observation exceeds the 1/F limit, the user should consider conferring with the local support staff, and maybe justify the tactics they will use to overcome the 1/F limit in their proposal.

The calculator uses the following estimates:

Table 1

System	Upper Limit for t_{Total}·BW
Mustang	10^5 MHz · s
Ka-Band with CCB	3.5×10^5 MHz · s
All Other Systems	See Mason (2013)

C. Confusion Limit

Desired sensitivities in some observing modes may not be reachable due to confusion within the beam from multiple background sources. The confusion limit depends upon the topocentric observing frequency (in MHz), the FWHM beam width of the telescope (in arc minutes) at that frequency, and the user's chosen units for sensitivity. Standard wisdom recommends that one stay under five times the confusion limit for a reliable detection. The calculator estimates the limit using the following equations (Condon 2002):

Table 2

Units	5x Confusion Limit
S (Jy)	$\frac{0.13 \cdot FWHM^2}{Frequency^{0.7}}$
T_R^*(K)	$\frac{0.13 \cdot \eta_S \cdot \pi \cdot (DishRadius \cdot FWHM)^2}{2k \cdot \eta_{fss} \cdot Frequency^{0.7}}$
T_{MB}(K)	$\frac{0.13 \cdot \eta_S \cdot \pi \cdot (DishRadius \cdot FWHM)^2}{2k \cdot \eta_{MB} \cdot Frequency^{0.7}}$
T_A(K)	$\frac{0.13 \cdot \eta_S \cdot \pi \cdot (DishRadius \cdot FWHM)^2}{2k \cdot Frequency^{0.7} \cdot AtmosAtten}$

If the user is entering a sensitivity to derive a t_{Total}, then the calculator warns the user whenever the sensitivity is smaller than the corresponding value from the above table. If the user enters a time in order to derive sensitivity, then the calculator warns the user whenever the calculated sensitivity is smaller than the corresponding value from the above table. In both cases, the warning presents the value of the confusion limit.

As with 1/F limit, the calculator cannot distinguish whether or not the specified observing tactics are or are not limited by confusion. Instead, the calculator assumes the user is best qualified to make that judgment.

D. Determining Values for Various Quantities

Many of the above equations depend upon values for EST_{TS} which in turn depends upon τ, air mass (i.e., elevation), receiver temperature, T_{Atm}, background source temperature, spillover and the cosmic microwave background as well as the yet-to-be defined quantities η_{Track} and η_{Surf}. The calculator, of course, doesn't know the details of the weather conditions or elevation ranges over which the observations will happen. Instead, it makes the assumption that the observations will happen under typical opacity and wind conditions, as determined by DSS simulations (Condon & Balser, 2011), for the chosen semester, topocentric frequency, and receiver. It also assumes observations will be taken symmetrically around the meridian to a frequency-dependent minimum elevation.

E.1 EST_{TS}

In various DSS memos, we define the Effective System temperature as $EST = T_{sys} \cdot AtmosAtten$. This is the system temperature one would use in the radiometer equation to get the same sensitivity as if the observations were taken above the atmosphere.

In addition to atmospheric loses, the calculator also uses estimates for the relative loss of efficiencies due to tracking or surface errors (η_{Track}, η_{Surf}, as defined in DSS memos) from winds or daytime observing. The quantity EST_{TS} used by the calculator is the EST with the extra losses due to tracking and surface errors.

$$EST_{TS} = \frac{EST}{\sqrt{\eta_{Track}\eta_{Surf}}} = \frac{T_{sys} \cdot AtmosAtten}{\sqrt{\eta_{Track}\eta_{Surf}}}$$

The DSS simulators provide the calculator with enough information that one can directly estimate a typical value for EST_{TS} for most observing setups. The DSS quantity η_{DSS} is an observing efficiency, with respect to time, that is normalized to the best conditions that are possible for the observing frequency, receiver, and source elevation. That is, a value of $\eta_{DSS} = 0.5$ suggests one will need twice as much observing time to achieve the same sensitivity as one would under the best opacity, wind, surface, … conditions. η_{DSS} is the product of the observing efficiency for atmospheric conditions (η_{Atm}), tracking errors due to the telescope's pointing accuracy under various wind conditions (η_{Track}) and surface errors (η_{Surf}).

By the DSS definition, $\eta_{Atm} = (EST_0/EST)^2$, where EST_0 is the effective system temperature for the best possible weather conditions (see §E.2). Thus:

$$\eta_{DSS} = \eta_{Track}\eta_{Surf}\left(\frac{EST_0}{EST}\right)^2 = \left(\frac{EST_0}{EST_{TC}}\right)^2$$

In most cases, one would use $EST_{TS} = EST_0/\sqrt{\eta_{DSS}}$. However, this will not be the case if one were observing toward a strong background source. To compensate for a background source with an effective black-body temperature of T_{BG}, one uses instead:

$$EST_{TS} = \frac{T_{BG}}{\sqrt{\eta_{Track}\eta_{\text{Surf}}}} + \frac{EST_0}{\sqrt{\eta_{DSS}}}$$

The DSS simulations have supplied values for η_{DSS} and the product $\eta_{Track}\ \eta_{Surf}$, all of which can be assumed to be frequency and receiver dependent. At all but the highest frequencies $\eta_{Track}\ \eta_{Surf} \approx 1$. The user either supplies T_{BG} or asks the calculator to provide an estimate of the galactic background, derived from the 408 MHz observations of Haslam et al (1981), for a specified Right Ascension, Declination and observing frequency.

E.2 EST_0

EST_0, the effective system temperature under the best weather conditions, is obtained from:

$$EST_0 = \left(T_{Rcvr} + T_{Spill} + T_{Atm}\right) \cdot e^{\tau_0 \cdot \text{AirMass}} - (T_{Atm} - T_{CMB})$$

where τ_0 is the best possible opacity at the observing frequency and AirMass is the ***typical*** air mass for the range of desired elevations. T_{Atm} is the equivalent black-body temperature of the atmosphere at the observing frequency for the best of weather conditions. The calculator assumes that both T_{Spill} (= 3 K) and T_{CMB} (=2.7 K) are receiver and frequency independent and that T_{Spill} is independent of elevation. T_{Rcvr} is receiver and frequency dependent and take on values provided by the receiver engineers. Since τ_0 and T_{Atm} are frequency and weather dependent, the calculator uses values derived from historical weather data averaged over five years year. (Due to the NRAO's semester scheduling system that begin and end in mid winter/summer, using yearly averages provides sufficiently accurate values.)

E.3 Atmospheric Attenuation (AtmosAtten)

Note that a number of equations require AtmosAtten, that is the expected, typical (not best) atmospheric attenuation under which one can expect to be scheduled. With a bit of algebra:

$$AtmosAtten = e^{\tau \cdot Airmass} = \frac{EST_0\sqrt{\frac{\eta_{Track}\eta_{\text{Surf}}}{\eta_{DSS}}} + (T_{Atm} - T_{CMB})}{T_{Rcvr} + T_{Spill} + T_{Atm}}$$

E.4 $\eta_l\,\eta_{fss}$, η_{MB}, and η_S

The aperture efficiency of the GBT (η_A) is based on an rms surface error of 220 µm at the rigging elevation near 45°, with worse surface errors at elevations away from 45°. Observations at 41 GHz show that one can adequately model the elevation-dependence of surface errors using a 2nd order polynomial model.

$$rms_{Surface} = (415.36 - 7.11 \cdot El + 0.0656 \cdot El^2)\ \mu\text{m}$$

The calculator uses the Ruze equation to determine both the elevation and frequency dependence of η_A :

$$\eta_A = \eta_0 \cdot \mathrm{e}^{-\left(4.19\times10^{-8}\cdot rms_{Surface}\cdot Frequency\right)^2}$$

The calculator assumes η_0 = 0.71 for the long-wavelength aperture efficiency. Since η_A varies significantly with elevation at the highest frequencies, the calculator uses numerical integration over elevation to derive an average η_A.

For extended sources, the calculator assumes a source with a uniform brightness distribution with a circular shape of diameter θ_{Source}. From models of the telescope's beam with a typical feed illumination pattern (taper) of -13 dB, we derive the following approximations:

$$\eta_S = \eta_A/(1 - 0.03740x + 0.2842x^2 - 0.1282x^3)$$
$$\eta_l\,\eta_{fss} = \eta_A/\left[-0.1192 + 0.9722/\left(1 - e^{-0.08568\cdot x^2}\right)\right]$$
$$\eta_{MB} = 1.16\eta_A$$

where $x = \theta_{Source}/FWHM$.

Note:

- The value of η_{MB} is independent of source size since, by definition, the source has a size that extends just to the first nulls in the antenna beam pattern (θ_{Source}~2.6 FWHM).
- As the source size decreases η_S goes to η_A while η_{fss} goes to zero.
- The calculator's upper limit for the source diameter that is to first null (~2.6 FWHM). Given the clean optics of the GBT, and the rough accuracy of the calculator, using T_{MB} is justifiable for very extended sources.
- When using the T_R^* temperature scale, the calculator does not allow the source size to be below 0.2 FWHM as T_R^* is not defined for a point source.

E.5 Average Air Mass and Suggested Minimum Elevation

The calculator needs to determine a typical air mass under which one can expect an observation to be run. The air mass is taken as a weighted average over the expected elevation range of an observation. It is sufficiently accurate for the sensitivity calculator to use csc(El) as an approximation for air mass.

From the source declination, the calculator determines the maximum elevation, which occurs at source transit (upper transit if the source is circumpolar). Using the algorithms in §3.4.2 of Condon & Basler (2011), the calculator derives a suggested minimum elevation that is based upon the source's elevation at transit and the topocentric observing frequency. The user need not use the suggested minimum and can choose a minimum elevation as long as it is above 5° (the limit for the GBT) or, if observing a circumpolar source, the source's elevation at lower transit.

The weighted average air mass is then calculated from the minimum and maximum elevations via:

$$AirMass = \frac{57.29 \cdot ln\left(\frac{\tan(El_{Max}/2)}{\tan(El_{Min}/2)}\right)}{El_{Max} - El_{Min}}$$

E.6 Opacity (τ)

Note that opacity is never directly used by the calculator (only AtmosAtten is used directly) but a value for opacity is supplied to help for the user in planning observations. The calculator provides an estimate of the expected opacity for the typical weather and the specified range in elevations:

$$\tau = \frac{ln(AtmosAtten)}{AirMass}$$

E.7 T_{SYS}

T_{SYS} is never directly used by the calculator but a value for T_{SYS} is supplied as a help for the user in planning observations. The calculator provides a typical T_{Sys} using the typical weather conditions:

$$T_{Sys} = \frac{EST_{TS} \cdot \eta_{Track} \cdot \eta_{Surf}}{AtmosAtten}$$

E.8 K_1 and K_2

Values for K_1 and K_2 are backend and backend mode dependent. For the GBT Spectrometer (ACS), the value of K_1 depends upon whether the observations are made in 3- or 9-levels modes. Only those observations that need the highest frequency or velocity resolutions should use 3-level sampling since it is far less efficient than 9-level sampling. The dividing point as to whether an observation should use 3- or 9-level depends upon the user's expected bandwidth after smoothing (BW) and the specified number of spectral windows ($N_{Windows}$) and feeds (N_{Feeds}) in the following way:

Table 3

GBT Spectrometer Mode	BW/($N_{Windows}$ N_{Feeds})	Sampling	K_1	K_2
200 or 800 MHz	Any	3-level	1.235	1.21
50 MHz	< 0.76 kHz	3-level	1.235	1.21
	>= 0.76 kHz	9-level	1.032	1.21
12.5 MHz	< 0.19 kHz	3-level	1.235	1.21
	>= 0.19 kHz	9-level	1.032	1.21

For the remaining backends, the calculator uses:

Table 4

Backend	K_1	K_2
Mustang, CCB, GUPPI, DCR, VEGAS, Zpectrometer	1	1
Spectral Processor	1.30	1.21

E.9 N_{Samp}^{Uncorr}

N_{Samp}^{Uncorr} represents the degree to which the data that are averaged together are uncorrelated. It describes how the sensitivity improves by averaging data from orthogonal polarizations or by use of different observing methods:

$$N_{Samp}^{Uncorr} = \mathrm{DualPol} \cdot \mathrm{ObervingMethod}$$

- DualPol = 2 if the user has specified they will be averaging polarizations, otherwise = 1.
- ObservingMethod = 2 if the user has specified in-band frequency switching and any of the 'nodding' observation types, otherwise = 1.

E.10 FWHM beam size

Although the FWHM beam width of the GBT depends upon the feed illumination pattern and observing frequency (Goldsmith, 1987, 2002), for all receivers covered by the calculator, it is sufficient to use a feed taper of 13 dB and a 50-m dish radius. Therefore, using the formulae of Goldsmith:

$$FWHM = \frac{12.3 \cdot 10^3}{\mathrm{Frequency}} \mathrm{arc\ minutes}$$

E.11 Topocentric frequency, frequency resolution, velocity resolution, and velocity coverage

The calculator provides values for the following quantities as a way to help observers plan their observations as well as to provide values it needs for various calculations.

1. To derive the ***highest*** topocentric frequency spacing the backend will allow, the calculator divides the bandpass width selected by the user by the maximum number of channels that the user's configuration will allow.
2. To derive an approximate topocentric frequency from the user-specified rest frequency and velocity or redshift:

Table 5

Velocity Def.	Topocentric Freq.
Radio	$RestFreq \cdot (1 - V/c)$
Optical	$RestFreq/(1 + V/c)$
Redshift	$RestFreq/(1 + z)$

The calculator can provide only an approximate topocentric frequency -- it has no knowledge of when an observation will take place and cannot estimate the the telescope motion with respect to a reference frame.

3. To convert a rest frame frequency resolution (Δf_{REST}) to a topocentric frequency resolution:

Table 6

Velocity Def.	Topocentric Freq. Resolution
Radio	$\Delta f_{REST} \cdot (1 - V/c)$
Optical	$\Delta f_{REST}/(1 + V/c)$
Redshift	$\Delta f_{REST}/(1 + z)$

4. To convert a rest frame velocity resolutions (ΔV_{REST}) to a topocentric frequency resolution:

Table 7

Velocity Def.	Topocentric Freq. Resolution
Radio	$RestFreq \cdot \Delta V_{REST}/c$
Optical	$\frac{RestFreq \cdot \Delta V_{REST}}{c \cdot (1 + V/c)^2}$
Redshift	$\frac{RestFreq \cdot \Delta V_{REST}}{c \cdot (1 + z)^2}$

The calculator checks that the user's topocentric frequency resolution is not smaller than K2 * highest topocentric frequency spacing the backend will allow, as calculated in step 1.

References

Baars, J.W.M, 1973, *IEEE Trans on Ant and Prop,* Vol AP-21, No. 4, p. 461.

Condon, J.J. and Balser, D.S., 2011, "Dynamic Scheduling Algorithms, Metrics, and Simulations", DS project Note 5.5, (https://safe.nrao.edu/wiki/pub/GB/Dynamic/DynamicProjectNotes/dspn5.5.pdf)

Goldsmith, P., 1987, Int Journal of Infrared and Millimeter Waves, Vol. 8, No. 7, p. 771.

Goldsmith, P, 2002, in *Single-Dish Radio Astronomy: Techniques and Applications*, ed. Stanimorivic, Altschuler, Goldsmith, and Salter (ASP, Vol. 278), p. 45.

Kutner, M.L. and Ulich, B.L., 1981, *Astrophysical Journal*, **250**, 341.

Mason, B. 2013, "GBT Receivers' Continuum Sensitivity", GBT Memo 282, (https://safe.nrao.edu/wiki/pub/GB/Knowledge/GBTMemos/GBT_Memo_282.pdf).